

p/adic LLC for  $GL_2(\mathbb{Q}_p)$ :

mod  $p$ :

$$\left. \begin{array}{l} \pi(\Gamma, 0) : \text{supersingular} \\ \text{with socle} \\ \text{Sym}^{\Gamma} \mathbb{F}^2 \\ \text{Sym}^{p-1-\Gamma} \mathbb{F}^2 \otimes \det^{\Gamma} \end{array} \right\} \longleftrightarrow \text{ind}_{G_{\mathbb{Q}_p^2}}^{G_{\mathbb{Q}_p}} (\omega_2^{\Gamma+1})$$

$$0 \rightarrow \text{ind}_{B(\mathbb{Q}_p)}^{GL_2(\mathbb{Q}_p)} \chi_2 \otimes \chi_1 \omega^{-1} \rightarrow * \rightarrow \text{ind}_{B(\mathbb{Q}_p)}^{GL_2(\mathbb{Q}_p)} \chi_1 \otimes \chi_2 \omega^{-1} \rightarrow 0 \longleftrightarrow [0 \rightarrow \chi_1 \rightarrow \bar{\rho} \rightarrow \chi_2 \rightarrow 0]$$

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if  $\chi_1 \chi_2^{-1} \notin \{1, \omega\}$

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socle:

if  $\chi_1 = \omega^{\Gamma+1}$  :  $\text{Sym}^{\Gamma} \mathbb{F}^2$

$\chi_2 = 1$

$\Gamma \in \{1, \dots, p-3\}$

$\otimes \begin{bmatrix} \omega & 0 \\ 0 & 1 \end{bmatrix} \text{Sym}^{p-3-\Gamma} \mathbb{F}^2 \otimes \det$

if  $*$  is split if  $\bar{\rho}$  is split

Serre weights:

$$W(\bar{\rho}) = \{ \text{Sym}^{\Gamma} \mathbb{F}^2, \text{Sym}^{p-3-\Gamma} \mathbb{F}^2 \otimes \det^{\Gamma+1} \}$$

if  $\bar{\rho}$  is split

Char 0 (crystalline case)

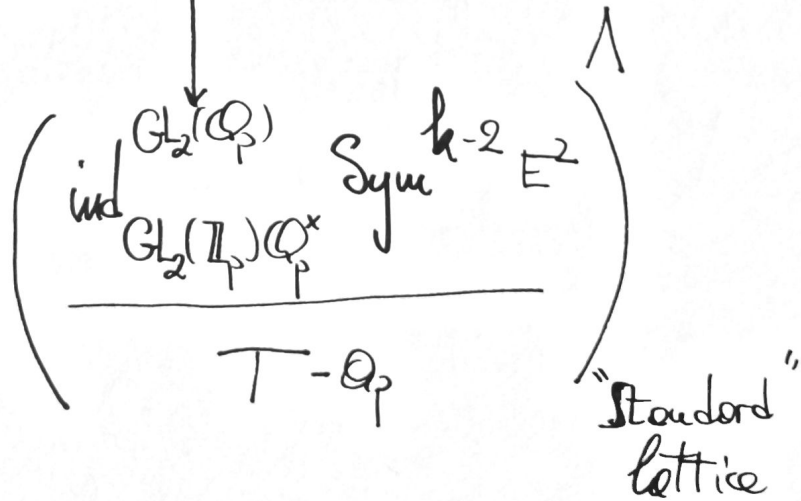
$k \geq 2, \alpha_p \in m_E$  must be  $\left\{ \begin{array}{l} \text{filtered } \varphi\text{-module } D_{k, \alpha_p} \\ \text{weakly admissible} \end{array} \right.$  over  $E$

Colmez-Fantaine

$V_{k, \alpha_p}$  crystalline, HT weight  $(0, k-1)$

$$\text{WD}(V_{k, \alpha_p}) = \mu_{\lambda_1} \oplus \mu_{\lambda_2}, \quad \begin{array}{l} \lambda_1 \cdot \lambda_2 = p^{k-1} \\ \lambda_1 + \lambda_2 = \alpha_p \end{array}$$

Breuil-Beggs



Berger :

$$\left( \frac{\text{ind}_{G_2(\mathbb{Z}_p)}^{G_2(\mathbb{Q}_p)} \text{Sym}^{k-2} \mathcal{O}_E}{T-\mathcal{O}_p} \right) \otimes_{\mathcal{O}_E} F \cong \left( \overline{V_{k, \mathcal{O}_p}} \right)^{\mathfrak{N}}$$

↓

$\rightarrow = \pi(k-2, 0) \text{ if } k \leq p+1$